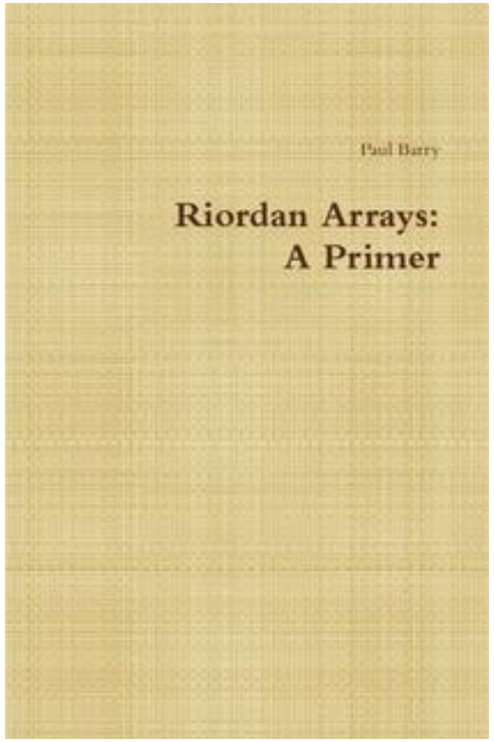
A brief history of Riordan arrays



From antiquity to today

Paul Barry

WIT 8/3/17

The binomial theorem

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

We recall that

$$\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a+b \\
(a+b)^2 &= a^2+2ab+b^2 \\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\
(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4
\end{aligned}$$

$$\cdots$$

$$
\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a+b \\
(a+b)^2 &= a^2+2ab+b^2 \\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\
(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4
\end{aligned}
$$

$$\cdots$$

Pascal's triangle

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 \\
1 & 3 & 3 & 1 & 0 & 0 & 0 \\
1 & 4 & 6 & 4 & 1 & 0 & 0 \\
1 & 5 & 10 & 10 & 5 & 1 & 0 \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{pmatrix}
$$

Blaise Pascal – France

1623 - 1662

$$
\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a+b \\
(a+b)^2 &= a^2+2ab+b^2 \\
(a+b)^3 &= a^3+3a^2b+3ab^2+b^3 \\
(a+b)^4 &= a^4+4a^3b+6a^2b^2+4ab^3+b^4 \\
&\cdots
\end{aligned}
$$

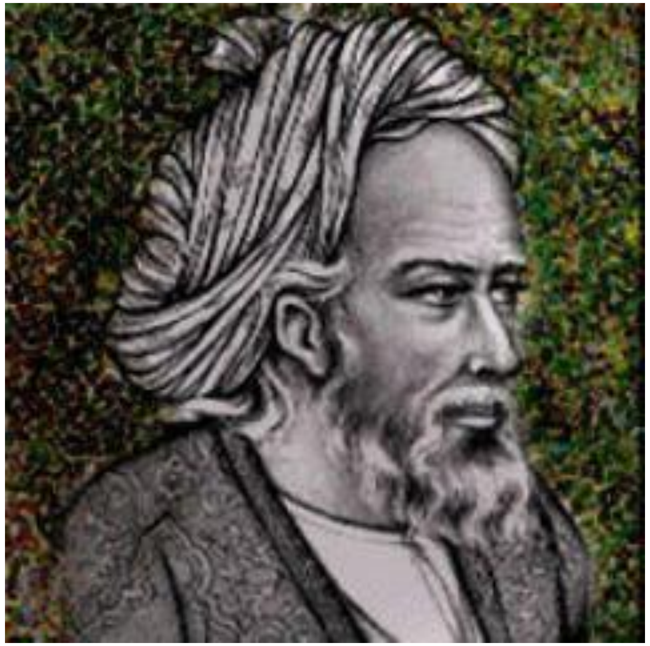
Euclid: Greece 2nd century BC

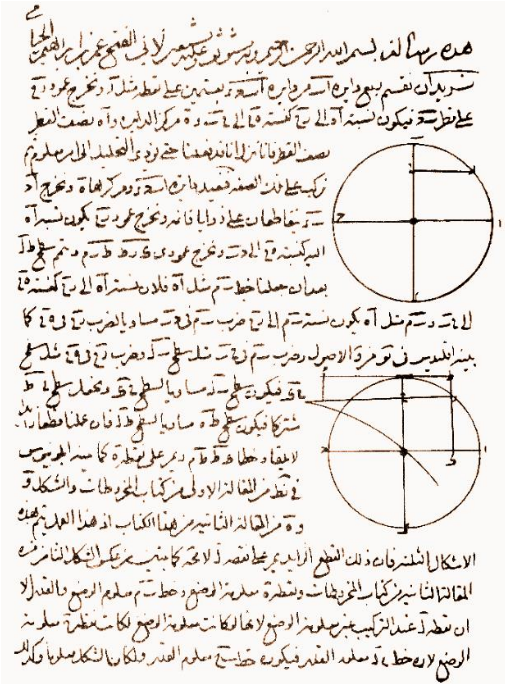
India: 6th century BC

Binomial coefficients, as combinatorial quantities expressing the number of ways of selecting k objects out of n without replacement, were of interest to the ancient Hindus. The earliest known reference to this combinatorial problem is the Chandaḥśāstra by the Hindu lyricist Pingala (c. 200 B.C.), which contains a method for its solution.

The binomial theorem as such can be found in the work of 11th-century Persian mathematician Al-Karaji, who described the triangular pattern of the binomial coefficients. He also provided a mathematical proof of both the binomial theorem and Pascal's triangle, using a primitive form of mathematical induction.

The Persian poet and mathematician Omar Khayyam (1048 – 1131) was probably familiar with the formula to higher orders, although many of his mathematical works are lost.





The Moving Finger writes; and, having writ,
Moves on: nor all thy Piety nor Wit,
Shall lure it back to cancel half a Line,
Nor all thy Tears wash out a Word of it.
But helpless pieces in the game He plays,
Upon this chequer-board of Nights and Days,
He hither and thither moves, and checks… and slays,
Then one by one, back in the Closet lays.
And, as the Cock crew, those who stood before
The Tavern shouted— "Open then the Door!
You know how little time we have to stay,
And once departed, may return no more."
A Book of Verses underneath the Bough,
A Jug of Wine, a Loaf of Bread—and Thou,
Beside me singing in the Wilderness,
And oh, Wilderness is Paradise enow.

The Moving Finger writes; and, having writ,
Moves on: nor all thy Piety nor Wit,
Shall lure it back to cancel half a Line,
Nor all thy Tears wash out a Word of it.

But helpless pieces in the game He plays,
Upon this chequer-board of Nights and Days,
He hither and thither moves, and checks… and slays,
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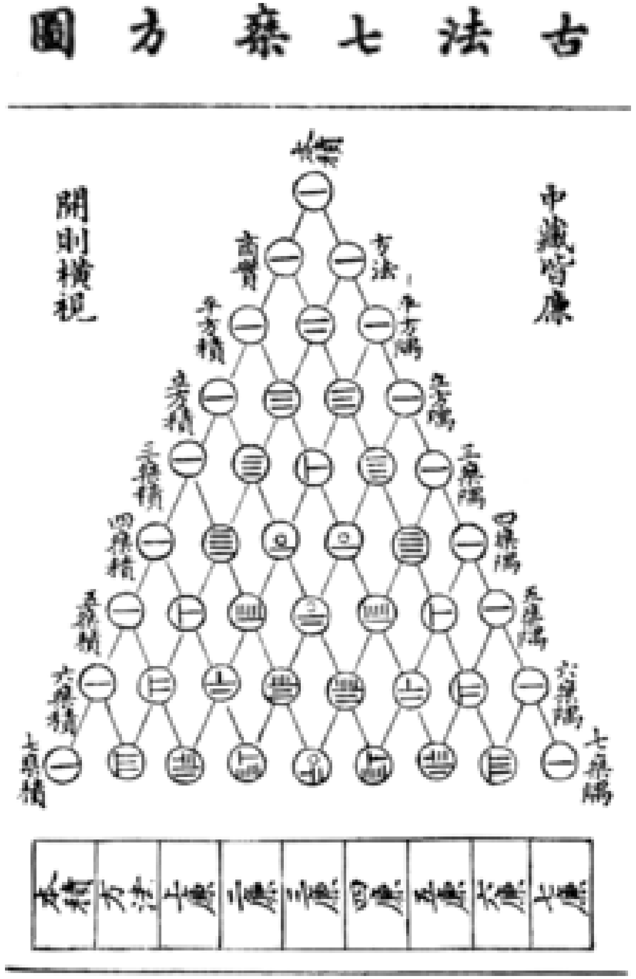
You know how little time we have to stay,
And once departed, may return no more."

A Book of Verses underneath the Bough,
A Jug of Wine, a Loaf of Bread—and Thou,
Beside me singing in the Wilderness,
And oh, Wilderness is Paradise enow.

The binomial expansions of small degrees were known in the 13th century mathematical works of Yang Hui and also Chu Shih-Chieh. Yang Hui attributes the method to a much earlier 11th century text of Jia Xian, although those writings are now also lost.

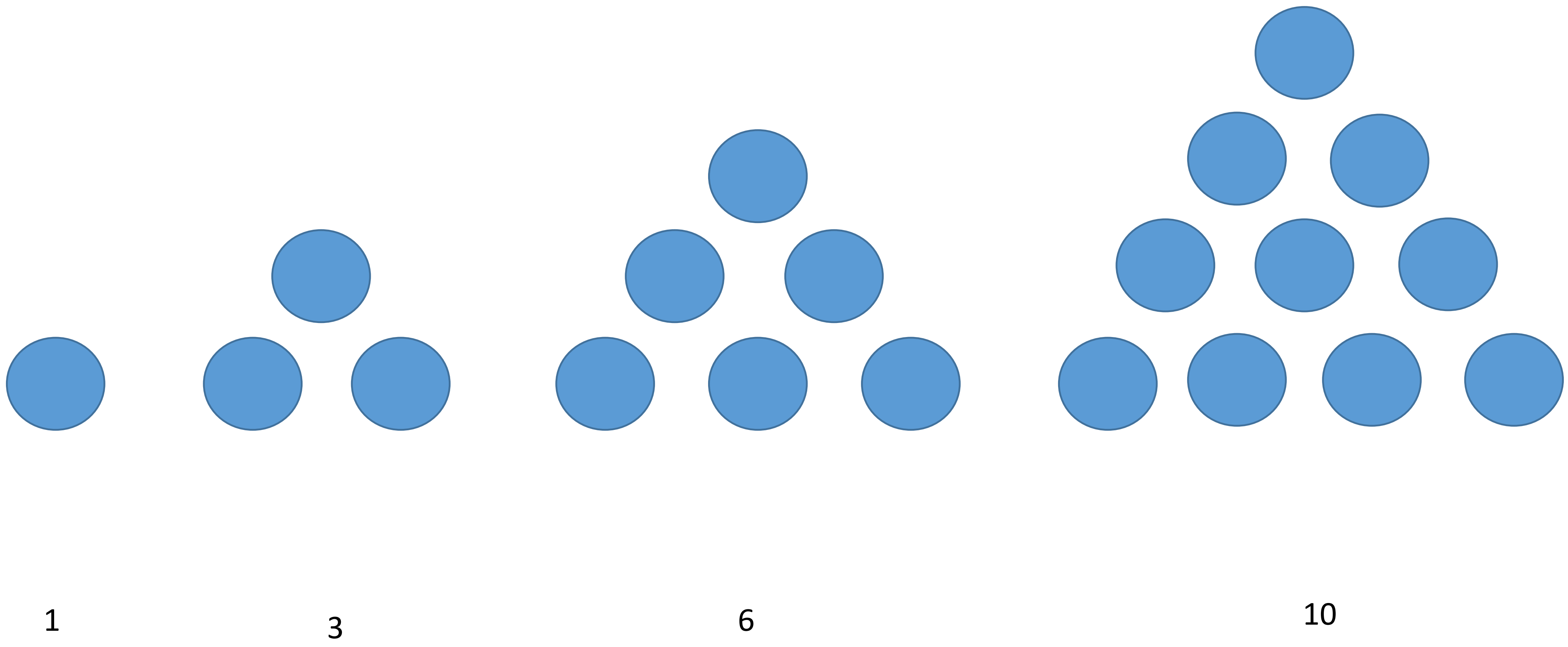


Figurate numbers

Pythagoreans
Theon of Smyrna
Nicomachus
2^{nd} century AD

Egypt: 3^{rd} century BC

$$0 + 1 + 2 + 3 + 4 + \cdots = \frac{n(n+1)}{2}$$



1	2	3	4	5	6	7	8	9	10
1	3	6	10	15					
1	4	10	20						
1	5	15	35						
1	6	21							

1	2	3	4	5	6	7	8	9	10
1	3	6	10	15	21	28	36	45	55
1	4	10	20	35	56	84	120	165	220
1	5	15	35	70	126	210	330	495	715
1	6	21	56	126	252	462	792	1287	2002

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10
1	3	6	10	15	21	28	36	45	55
1	4	10	20	35	56	84	120	165	220
1	5	15	35	70	126	210	330	495	715
1	6	21	56	126	252	462	792	1287	2002

Lattice paths

♖	1	1	1
1	2	3	4
1	3	6	10
1	4	10	20

Pascal's triangle overlaid on a square grid gives the number of distinct paths to each square, assuming only rightward and downward movements are considered.

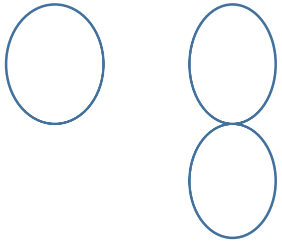
Defining patterns

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 \\ 1 & 4 & \bigcirc\!\!\!\!\!\!\;6 & \bigcirc\!\!\!\!\!\!\;4 & 1 & 0 & 0 \\ 1 & 5 & 10 & \bigcirc\!\!\!\!\!\!\!\;10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & \textcircled{2} & \textcircled{1} & 0 & 0 & 0 & 0 \\ 1 & 3 & \textcircled{3} & 1 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}$$



1 x + 1 x

=

Let P =

$$
P = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 \\
1 & 3 & 3 & 1 & 0 & 0 & 0 \\
1 & 4 & 6 & 4 & 1 & 0 & 0 \\
1 & 5 & 10 & 10 & 5 & 1 & 0 \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{pmatrix}
$$

$$
P^{-1}\bar{P} = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 & 0 \\ 6 & 10 & 5 & 1 & 0 & 0 & 0 \\ 22 & 38 & 22 & 7 & 1 & 0 & 0 \\ 90 & 158 & 98 & 38 & 9 & 1 & 0 \\ 394 & 698 & 450 & 194 & 58 & 11 & 1 \end{pmatrix}$$

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\boxed{{}^{1}2} & \boxed{{}^{1}3} & \boxed{{}^{1}1} & \boxed{{}^{1}0} & \boxed{{}^{1}0} & \boxed{{}^{1}0} & \boxed{{}^{1}0} \\
\boxed{6} & 10 & 5 & 1 & 0 & 0 & 0 \\
22 & 38 & 22 & 7 & 1 & 0 & 0 \\
90 & \boxed{{}^{1}158} & \boxed{{}^{2}98} & \boxed{{}^{2}38} & \boxed{{}^{2}9} & \boxed{{}^{2}1} & \boxed{{}^{2}0} \\
394 & 698 & \boxed{450} & 194 & 58 & 11 & 1
\end{pmatrix}
$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 2 & 1 & 0 \\ 1 & 2 & 2 & 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Generating functions

We carry out the long division of 1 by 1-x to get 1/(1-x)

$$
\begin{array}{r}
1 + x + x^2 + x^3 + \cdots \\
1 - x \overline{)\,1 \qquad\qquad\qquad\qquad\quad} \\
\underline{1 - x} \qquad\qquad\qquad\qquad \\
x \qquad\qquad\qquad\quad \\
\underline{x - x^2} \qquad\qquad\quad \\
x^2 \qquad\qquad \\
\underline{x^2 - x^3} \qquad \\
x^3 \quad \\
\underline{x^3 - x^4} \\
x^4 \cdots
\end{array}
$$

and so we can write

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots .$$

We have

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = 1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + 1 \cdot x^4 + \cdots .$$

and so we say that 1/(1-x) generates the sequence 1, 1, 1, 1, 1, …..

We use the following mathematical shorthand notation

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

In general, if we have

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

then we say that f(x) generates the sequence a_n and that *f*(x) is the generating function of a_n

What is the generating function of the sequence 1, 2, 3, 4, 5, …. ?

What is the generating function of the sequence 1, 2, 3, 4, 5, …. ?

We carry out the long division of 1/(1-x) by 1-x. Remember that we can write

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = 1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + 1 \cdot x^4 + \cdots .$$

$$
\begin{array}{r l}
 & 1 + 2x + 3x^2 + 4x^3 + \cdots \\
1 - x \,\big) & 1 + x + x^2 + x^3 + \cdots \\
 & \underline{1 - x} \\
 & \quad 2x + x^2 + \cdots \\
 & \quad \underline{2x - 2x^2} \\
 & \qquad 3x^2 + x^3 + \cdots \\
 & \qquad \underline{3x^2 - 3x^3} \\
 & \qquad\quad 4x^3 + x^4 + \cdots \\
 & \qquad\quad \underline{4x^3 - 4x^4} \\
 & \qquad\qquad 5x^4 + x^5 + \cdots
\end{array}
$$

with answer given by

$$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots .$$

Thus we get the generating function

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots .$$

$$
\begin{array}{r l}
 & 1 + 3x + 6x^2 + 10x^3 + \cdots \\
1 - x \,\big) & 1 + 2x + 3x^2 + 4x^3 + \cdots \\
 & \underline{1 - x} \\
 & \quad 3x + 3x^2 + \cdots \\
 & \quad \underline{3x - 3x^2} \\
 & \qquad 6x^2 + 4x^3 + \cdots \\
 & \qquad \underline{6x^2 - 6x^3} \\
 & \qquad\quad 10x^3 + 5x^4 + \cdots \\
 & \qquad\quad \underline{10x^3 - 10x^4} \\
 & \qquad\qquad 15x^4 + 6x^5 + \cdots \; .
\end{array}
$$

Thus we get the generating function

$$\frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots$$

$$
\begin{aligned}
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots \\
\frac{x}{(1-x)^2} &= x + 2x^2 + 3x^3 + 4x^4 + 5x^4 + \cdots \\
\frac{x^2}{(1-x)^3} &= x^2 + 3x^3 + 6x^4 + 10x^5 + \cdots \\
\frac{x^3}{(1-x)^4} &= x^3 + 4x^4 + 10x^5 + \cdots \\
\ldots &= \ldots
\end{aligned}
$$

Now note that

$$
\frac{x}{(1-x)^2} = 0 + 1x + 2x^2 + 3x^2 + 4x^3 + \cdots
$$

$$
\frac{x^2}{(1-x)^3} = 0 + 0x + 1x^2 + 3x^3 + 6x^4 + \cdots
$$

The first column of Pacal's triangle is defined by $\frac{1}{1-x}$.

The second column is generated by

$$\frac{1}{1-x}\frac{x}{1-x}=\frac{x}{(1-x)^2}.$$

The third column is generated by

$$\frac{1}{1-x}\left(\frac{x}{1-x}\right)^2=\frac{x^2}{(1-x)^3}.$$

The fourth column is generated by

$$\frac{1}{1-x}\left(\frac{x}{1-x}\right)^3=\frac{x^3}{(1-x)^4}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}$$

$$\frac{1}{1-x} \quad \frac{x}{(1-x)^2} \quad \frac{x^2}{(1-x)^3} \quad \frac{x^3}{(1-x)^4} \quad \frac{x^4}{(1-x)^5} \quad \frac{x^5}{(1-x)^6} \quad \frac{x^6}{(1-x)^7}$$

Let $g(x) = \frac{1}{1-x}$ and $f(x) = \frac{x}{1-x}$. The first column is generated by $g(x)$. The second column is generated by

$$g(x)f(x).$$

The third column is generated by

$$g(x)f(x)^2$$

The fourth column is generated by

$$g(x)f(x)^3$$

A Riordan array R is defined by two power series

$$g(x) = \sum_{n=0}^{\infty} g_n x^n \quad \text{and} \quad f(x) = \sum_{n=1}^{\infty} f_n x^n$$

such that the *k*-th column of R is generated by

$$g(x) f(x)^k$$

We denote the Riordan array defined by $g(x)$ and $f(x)$ by $(g(x), f(x))$.

Thus Pascal's triangle is represented by

$$\left(\frac{1}{1-x}, \frac{x}{1-x}\right)$$

Lou Shapiro

Discrete Applied Mathematics 34 (1991) 229–239 229
North-Holland

The Riordan group

Louis W. Shapiro, Seyoum Getu, Wen-Jin Woan and
Leon C. Woodson

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Received 8 June 1989
Revised 4 November 1989

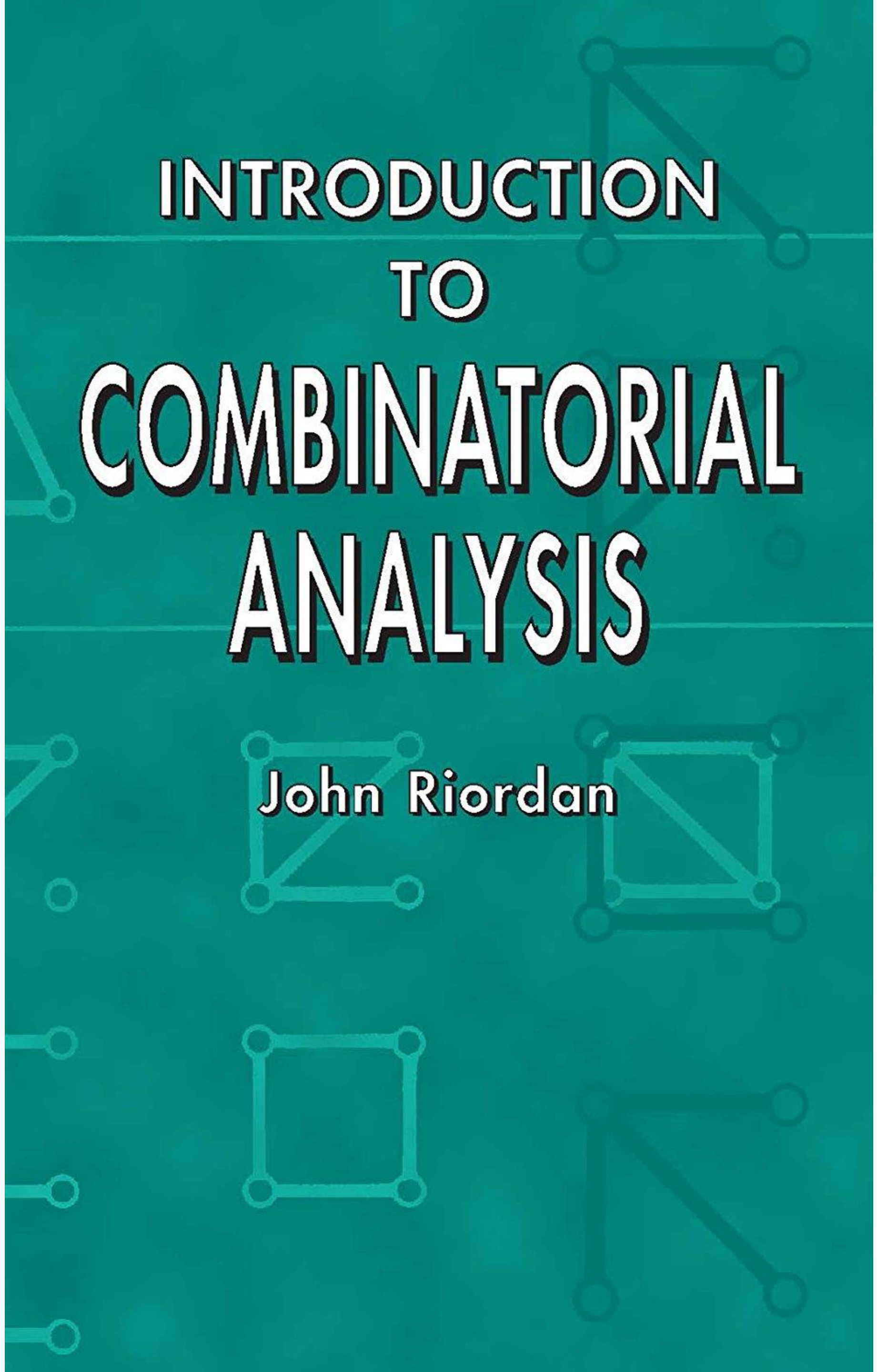
Abstract

Shapiro, L.W., S. Getu, W.-J. Woan and L.C. Woodson. The Riordan group, Discrete Applied Mathematics 34 (1991) 229–239.

Introduction

The central concept in this article is a group which we call the Riordan group. With the recent death of John Riordan this seems appropriate to name after him.

John Riordan 1903 - 1988



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Discrete Mathematics 22 (1978) 301–310.

PASCAL TRIANGLES, CATALAN NUMBERS AND RENEWAL ARRAYS

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Received 25 October 1976
Revised 6 June 1977

In response to some recent questions of L.W. Shapiro, we develop a theory of triangular arrays, called renewal arrays, which have arithmetic properties similar to those of Pascal's triangle. The Lagrange inversion formula has an important place in this theory and there is a close relation between it and the theory of renewal sequences. By way of illustration, we give several examples of renewal arrays of combinatorial interest, including complete generalizations of the familiar Pascal triangle and sequence of Catalan numbers.

1. Introduction

The binomial coefficients are as fundamental in combinatorial theory as they

The set of Riordan arrays is a group. The identity is (1, x).

You can multiply Riordan arrays to get Riordan arrays. You can "divide" a Riordan array by a Riordan array to get a Riordan array.

$$(g(x), f(x)) \cdot (u(x), v(x)) = (g(x)u(f(x)), v(f(x)))$$

$$(g(x), f(x))^{-1} = \left(\frac{1}{g(\bar{f}(x))}, \bar{f}(x)\right)$$

Here, $\bar{f}(x)$ is the *reversion* of the power series $f(x)$. It is the solution to the equation

$$f(u) = x$$

such that $u(0) = 0$.

Let us find the reversion of $\frac{x}{1-x}$. We must solve

$$\frac{u}{1-u} = x$$

This gives us

$$u = x - xu \implies u + ux = x$$

Thus $u(x) = \bar{f}(x) = \frac{x}{1+x}$.

We find that the inverse of Pascal's triangle, as a Riordan array, is given by

$$\left(\frac{1}{1-x}, \frac{x}{1-x}\right)^{-1} = \left(\frac{1}{1+x}, \frac{x}{1+x}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 1 & 5 & 10 & 10 & 5 & 1 & 0 \\ 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -3 & 1 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ -1 & 5 & -10 & 10 & -5 & 1 & 0 \\ 1 & -6 & 15 & -20 & 15 & -6 & 1 \end{pmatrix}$$

oeis.org/search?q=1%2C1%2C2%2C5%2C14%2C42&sort=&language=english&go=Search

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THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

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1,1,2,5,14,42 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

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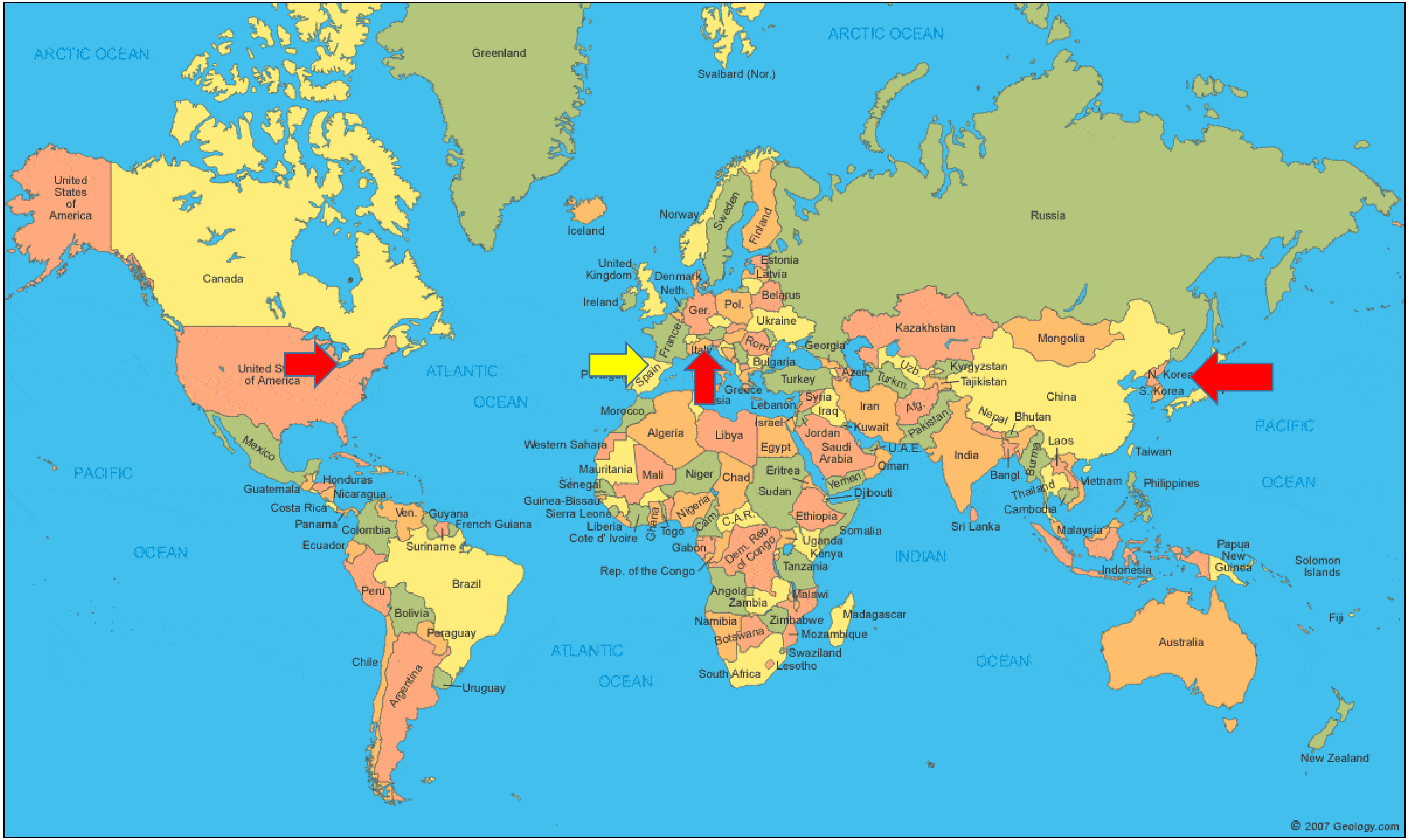
Sort: relevance | references | number | modified | created Format: long | short | data

A000108 Catalan numbers: C(n) = binomial(2n,n)/(n+1) = (2n)!/(n!(n+1)!). Also called Segner numbers. +20 2846

(Formerly M1459 N0577)

1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845,

Riordan array conferences 2014 - 2017



New York Times blog
https://wordplay.blogs.nytimes.com/2014/10/06/icosian/?_r=0

Mentions:

John Riordan
Neil Sloane
W. R. Hamilton
&
Padraig Kirwan
Kieran Murphy

Hamilton's Icosian Game

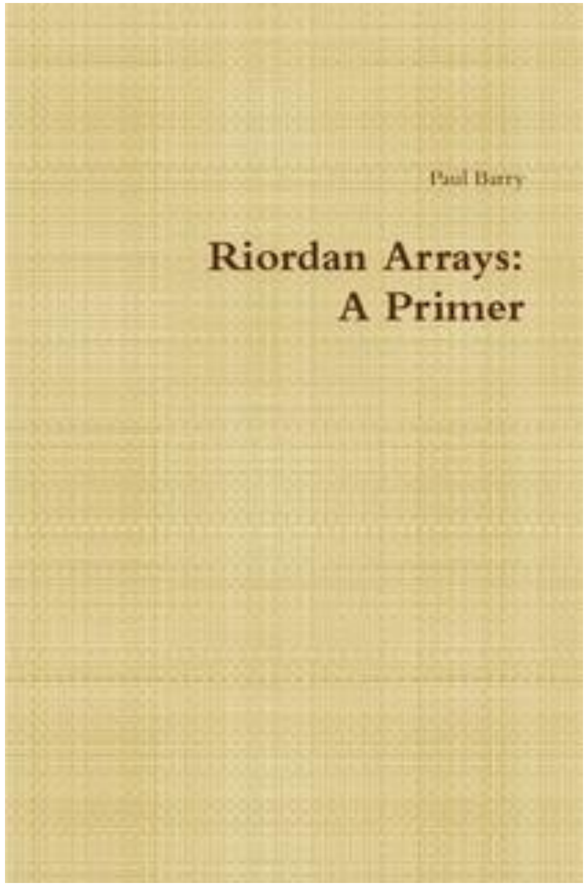
By GARY ANTONICK OCTOBER 6, 2014 12:00 PM 17

The River Liffey in Dublin, lifelong home of William Rowan Hamilton, Ireland's greatest mathematician.
Lonnie Schlein/The New York Times

Email

NUM
+BER
PLAY

This week's puzzle was suggested by Pádraig Kirwan, a professor of mathematics at Ireland's Waterford Institute of Technology, to commemorate an act of



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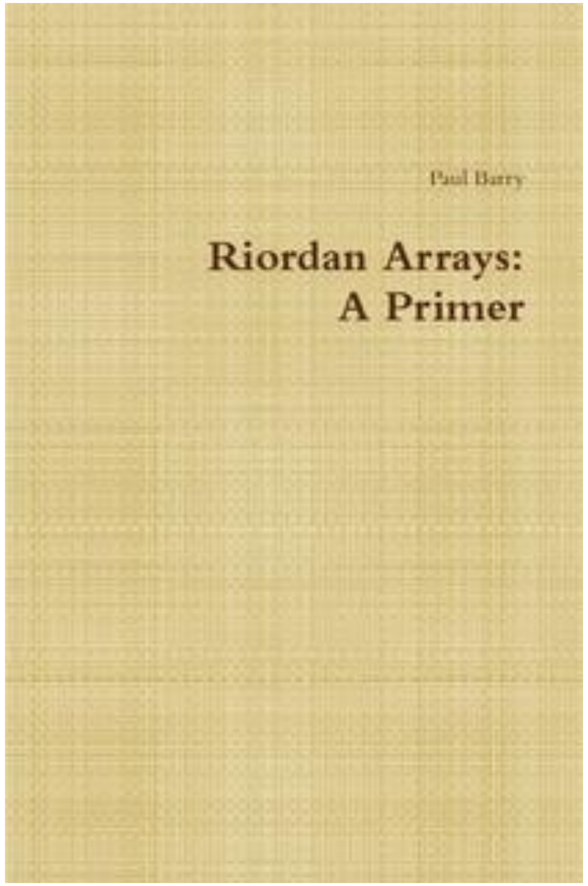
Thank you !



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